Review Final Questions:-

(1) Prove that the following argument valid:-

$\neg p \to r \wedge \neg s \equiv p \vee (r \wedge \neg s)$

$t \to s \equiv \neg t \vee s$

$u \to \neg p \equiv \neg u \vee \neg p$

$\neg w$

$\underline{u \vee w}$

$\therefore \neg t$

① $\cancel{\neg u} \vee \neg p$

$\underline{\cancel{u} \vee w}$

$\therefore \neg p \vee w$

② $p \vee (r \wedge \neg s)$

$\underline{\neg p \vee w}$

$\therefore (r \wedge \neg s) \vee w$

③ $(r \wedge \neg s) \vee w$

$\underline{\neg w}$

$\therefore r \wedge \neg s$

④ $\underline{r \wedge \neg s}$

$\therefore \neg s$

⑤ $\neg t \vee s$

$\underline{\neg s}$

$\therefore \neg t$ $\therefore$ valid

(2) Define $f : \mathbb{Z} \to \mathbb{Z}^+$ by $f(x) = |x| + 1$

(a) Find the Domain, co-Domain and Range:-

Domain: $\mathbb{Z}$, Co-Domain: $\mathbb{Z}^+$

Range: $f(a) = b \to |a| + 1 = b$

Solve for a:- $|a| = b - 1$

Range $= \{ K + 1 \mid K \in \mathbb{Z}^+ \}$

(b) Is f onto? I f one-to-one? Justify your answer :-

Is f onto? $f(a) = b \to |a| + 1 = b \to$ solve for $a \to |a| = b - 1$

b is ether K or $K+1$

when $b = K = 1$ $|a| = 1 - 1 = 0 \notin \mathbb{Z}^+$ $\therefore$ not onto

Is f one-to-one? $f(a_1) = f(a_2)$

$|a_1| + 1 = |a_2| + 1 \to |a_1| = |a_2|$

when $a_1 = 1$ & $a_2 = -1 \to |a_1| = |a_2|$ but $a_1 \neq a_2$ $\therefore$ not one-to-one

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(3) Prove that if x is a real number and $x^2 - x - 2 > 0$, then $x < -1$ or $x > 2$

Assume $x^2 - x - 2 > 0$ «Direct proof»

$(x-2)(x+1) > 0$

$x - 2 > 0$ or $x + 1 > 0$

$x > 2$ or $x < -1$

(4) let $A = \mathbb{Z}$ and define as:-

$\forall a, b \in A : aRb \iff 5a - 2b = 3n \ , n \in \mathbb{Z}$

(a) Show that R is an Equivalence relation

* Reflexive: $\forall x \in A : xRx$

$\in \mathbb{Z}$ $\quad 5x - 2x = 3k$

$\therefore 3x = 3K_1 \ , K_1 \in \mathbb{Z}$

* Symmetric: $\forall x, y \in A : xRy \longrightarrow yRx$

$\in \mathbb{Z}$ $\quad 5x - 2y = 3K_1 \quad 5y - 2x = 3K_2$

Suppose $5x - 2y = 3K_1 \ , K_1 \in \mathbb{Z}$

$2y = 5x - 3K_1 \Rightarrow y = \frac{5x - 3K_1}{2}$

subsit in $5y - 2x$

$5\left(\frac{5x - 3K_1}{2}\right) - 2x \longrightarrow \frac{25x - 3K_1}{2} - 2x \longrightarrow \frac{21x - 3K_1}{2} \longrightarrow 3\left(\frac{7x - K_1}{2}\right) = 3K_2 \ , K_2 \in \mathbb{Z}$

* Transitive: $\forall x, y, z \in A : xRy \wedge yRz \longrightarrow xRz$

$5x - 2y = 3K_1 \quad 5y - 2z = 3K_2 \quad 5x - 2z = 3K_3$

Suppose $5x - 2y = 3K_1 \ , K_1 \in \mathbb{Z} \longrightarrow (1)$ and

$5y - 2z = 3K_2 \ , K_2 \in \mathbb{Z}$

Add (1) and (2):-

$5x + 3y - 2z = 3K_1 + 3K_2 \longrightarrow 5x - 2z = 3(K_1 + K_2 - y) \longrightarrow 5x - 2z = 3K_3 \ , K_3 \in \mathbb{Z}$

(b) List 4 elements of [1]

$[a] = \{x \in A \mid xRa\}$

$[1] = \{x \mid xR1\}$

$= \{\ldots, \underbrace{1, 4, 7, 10}_{\text{4 elements}}, \ldots\}$